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ACTIVE DECELERATION OF ROTATIONAL MOTIONS OF A DYNAMICALLY ASYMMETRIC QUASIRIGID BODY

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In this article a minimum-time problem of deceleration of rotations of a free rigid body is studied analytically and numerically. It is assumed that a body contains a spherical cavity filled with highly viscous fluid. The body is subjected to a retarding torque of viscous friction. It is assumed that such body is dynamically asymmetric. An optimal control law for deceleration of rotations of the body is synthesized, and the corresponding time and phase trajectories are determined.

The asymptotic approach made it possible to determine the control evolutions of the magnitude squared of the elliptic functions modulus k2, dimensionless kinetic energy and kinetic moment. The qualitative properties of the optimal motion were also found.

The obtained results allow us to build a synthesis of the optimal deceleration of rotations of satellites and spacecrafts. They can be used to analyze dynamics of controlled spacecrafts.

Keywords: optimal deceleration, resistive medium, asymmetric body, cavity.

1. INTRODUCTION

Analysis of the motion of hybrid systems (i.e., objects containing elements with distributed and concentrated parameters) is of interest both theoretically and practically. This analysis can be done within the framework of the theory of singularly perturbed problems. Important results were obtained for systems containing quasi-rigid bodies. Combined rotational and translational motions of these systems are close (under certain conditions) to the motion of absolutely rigid bodies. The influence of non-ideal features are reduced to the effects of the temporal boundary layer type and to additional perturbing moments in the Euler equations of angular motion of a fictitious rigid body after the completion of transient processes. The analysis of motions of a rigid body with a cavity filled with a viscous fluid and in a resistive medium had received much attention [1–6]. The control of rotations of quasi-rigid bodies using concentrated (applied to the frame) torques received less attention. Researchers managed to distinguish a class of systems leading to smooth controls making it possible to apply singular disturbance methods without accumulation of boundary layer type errors appearing in the case of discontinuous (for example, bang-bang) controls [7–9]. In this paper, we investigate the problem of time-optimal deceleration of rotations of a dynamically non-symmetric body with a spherical cavity filled with highly viscous fluid (for small Reynolds numbers). In addition, the rigid body is subjected to the action of a small retarding torque of linear resistance of the medium. The rotations are controlled by a bounded torque, which can be exerted by vernier jet engines [7]. The model under consideration generalizes the results obtained earlier in [7–11]. The problem of optimal deceleration of rotations of a dynamically symmetric body containing a viscous–elastic element and a cavity filled with fluid is studied in [8]. The problem of time-optimal deceleration of rotations of a dynamically symmetric rigid body with a spherical cavity filled with highly viscous fluid and a moving mass attached to the body by an elastic joint with quadratic dissipation is considered in [9]. The problem of optimal deceleration of rotations of a dynamically symmetric body with a cavity filled with highly viscous fluid is considered in [10], where the rigid body is subjected to a small torque of viscous friction of the external medium. The problem of time-optimal deceleration of rotations of a dynamically asymmetric body in a resistive medium is considered in [11]. Approximate solutions of perturbed problems of time-optimal deceleration of rotations of rigid bodies about the center of mass (including objects with internal degrees of freedom) with applications to the spacecraft and aircraft dynamics were obtained in the monograph [7]. There, the deceleration of bodies having a cavity with viscous fluid was studied. The cases of axisymmetric and asymmetric (in the undisturbed state) bodies with a spherical cavity filled with highly viscous fluid were considered. The deceleration of perturbed rotations of a rigid body close to a spherically symmetric one under the action of the torque exerted by the linear resistance of the medium was analyzed.

2. STATEMENT OF THE PROBLEM

2.1 The equations of controlled rotations

We consider a dynamically nonsymmetric rigid body with moments of inertia satisfying, for definiteness, the inequalities $A_1 > A_2 > A_3$. Based on the approach described in [7], the equations of controlled rotations projected on the axes of the body-related coordinate system (the Euler equations) can be expressed as [1, 4, 5, 7]

$$J\dot{\omega} + [\dot{\omega} \times J\dot{\omega}] = \mathbf{M}^u + \mathbf{M}^r + \mathbf{M}^c \qquad (1)$$

Here, $\boldsymbol{\omega} = (p, q, r)$ is the vector of absolute angular velocity, $J = \mathrm{diag}\,(A_1, A_2, A_3)$ is the tensor of body inertia, $\mathbf{M}^u$ is the vector of control torque, $\mathbf{M}^r$ is the dissipation torque, and $\mathbf{M}^c$ is the torque of viscous fluid in the body cavity. The kinetic moment of the body is determined in the standard way as

$$\mathbf{G} = J\boldsymbol{\omega}\,,\ \mathbf{G} = (G_1, G_2, G_3)\,,$$

$$G_1 = A_1 p\,,\ G_2 = A_2 q\,,\ G_3 = A_3 r\,,$$

where $G = (G_1^2 + G_2^2 + G_3^2)^{1/2}$ is its magnitude.

To simplify the problem, we introduce structural constraints into system (1); in particular, we assume that the feasible values of the control torque $\mathbf{M}^u$ belong to a sphere [7]). This assumption is not inconsistent with the mass distribution and shape of the rigid body and is often used in attitude control problems. It is also believed that the diagonal tensor of the external resistance torque is proportional to the moment of inertia tensor; i.e., the dissipation torque is proportional to the kinetic moment

$$\mathbf{M}^r = -\lambda J\boldsymbol{\omega}\,. \qquad (2)$$

Here, λ is a constant coefficient depending on the medium properties. The resistance acting on the body is represented by a pair of forces. In this case, the magnitude of projections of the moment of this pair on the major axes of body inertia are $\lambda A_1 p$, $\lambda A_2 q$, $\lambda A_3 r$ and [4, 5]. This assumption is not contradictory.

Next, we assume that the cavity is filled with highly viscous fluid; i.e., $\upsilon >> 1$ ($\upsilon^{-1} \sim \varepsilon << 1$), where υ is the kinematic viscosity. The shape of the cavity is supposed to be almost spherical; then, following [1], for the tensor **P** of the viscous forces, we have

$$\mathbf{P} = P\mathrm{diag}(1,1,1)\,,\, P = 8\pi\rho a^7 / (525\vartheta)\,, \qquad (3)$$

where ρ, υ are the fluid density and kinematic viscosity, respectively; and a is the cavity radius. The tensor **P**, which depends only on the cavity shape, characterizes the internal dissipative torque in the quasi-static approximation due to the viscous fluid in the cavity. For simplicity, Eqs. (1) use the so-called scalar tensor defined by a single scalar $P > 0$. The components of this tensor have the form $\mathrm{P}_{ij} = P\delta_{ij}$, where δ_{ij} are the Kronecker symbols (the tensor **P** has this form if the cavity is spherical, for example). If the cavity is significantly nonspherical, there are considerable difficulties in determining the tensor components.

The admissible values of the moment $\mathbf{M}^u$ of the control forces are assumed to be bounded by the sphere

$$\mathbf{M}^u = b\mathbf{u}\,, |\mathbf{u}| \le 1\,; b = b(t, \boldsymbol{\omega})\,, \qquad (4)$$
$$0 < b_* \le b \le b^* < \infty\,,$$

where b is a scalar function bounded in the domain of variation of its arguments t and $\boldsymbol{G}$ according to conditions (4). This domain is given a priori or can be estimated from the initial data for $\boldsymbol{G}$ ($\boldsymbol{G}(t_0) = \boldsymbol{G}_0$) by integrating Eq. (1) with respect to $\boldsymbol{\omega}$. Below, we suppose that $b = b\,(t, \boldsymbol{G})$ (or $b = b(t)$ or $b = \mathrm{const}$).

2.2 The problem of time-optimal deceleration

We pose the problem of time-optimal deceleration of rotations

$$\boldsymbol{\omega}(t_0) = \boldsymbol{\omega}^0\,,\ \boldsymbol{\omega}(T) = 0\,,\ T \to \min_{\mathbf{u}}\,,\ |\mathbf{u}| \le 1\,. \qquad (5)$$

It is required to find an optimal control $u = u(t, \boldsymbol{\omega})$, the corresponding trajectory $\boldsymbol{\omega}(t, t_0, \boldsymbol{\omega}_0)$, the time $T = \mathrm{T}(t_0, \boldsymbol{\omega}_0)$, and the Bellman function $W = T(t, \boldsymbol{\omega})$. Based on dynamic programming and the Schwarz inequality, under the simplifying condition on the coefficient b ($b = b(t, \boldsymbol{G}) = b_0(t, \boldsymbol{G})$, where the zero subscript will be omitted below) a time-optimal control is constructed in the form (see [7])

$$M_p = -bA_1 pG^{-1}\,,\ M_q = -bA_1 qG^{-1}\,, \qquad (6)$$

$$M_r = -bA_3 rG^{-1}\,,\ b = b(t, G)\,.$$

With regard to external force factors, the torque of viscous fluid in the cavity $\mathbf{M}^c$ is determined as (see [1])

$$\mathbf{M}^c = \frac{P\rho}{\nu}\begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix}, \qquad (7)$$

where

$$m_1 = p\left(\lambda^2 + \frac{b^2}{G^2}\right) + \frac{2\lambda b}{G}p + \frac{1}{A_1}\left(\lambda + \frac{b}{G}\right)\times$$
$$\times\left(3qr\left(A_3 - A_2\right) + \frac{G\alpha_{33}}{1-\alpha_{33}^2}q\left(\alpha_{31}^2 + \alpha_{32}^2\right) - Gr\alpha_{32}\right) +$$
$$+\frac{p}{A_1 A_2 A_3}\left[q^2 A_2 (A_1 - A_2)(A_2 - A_3 + A_1) + \right.$$
$$\left. + r^2 A_3 (A_1 - A_3)(A_3 - A_2 + A_1)\right].$$

The expressions for m_2 and m_3 are obtained from m_1 in (7) by a cyclic permutation of A_1, A_2, A_3 and p, q, r. The coefficients $\lambda^2 + b^2/G^2$, $\lambda + b/G$, and $2\lambda b/G$ in m_i ($i = 1, 2, 3$) remain unchanged, and the terms containing α_{31}, α_{32}, $\alpha_{31}{}^2 + \alpha_{32}{}^2$ have a similar form. The direction cosines α_{ij} are expressed in terms of the Euler angles φ, ψ, and θ according to well-known formulas [12]. Neglecting the influence of $\mathbf{M}^u$ and $\mathbf{M}^r$ on $\mathbf{M}^c$, we obtain the torque of viscous fluid in the cavity in the form

$$\mathbf{M}^c = \frac{P}{A_1 A_2 A_3}\times$$
$$\times\begin{pmatrix} p\begin{bmatrix} q^2 A_2 (A_1 - A_2)(A_2 - A_3 + A_1) + \\ + r^2 A_3 (A_1 - A_3)(A_3 - A_2 + A_1) \end{bmatrix} \\ q\begin{bmatrix} r^2 A_3 (A_2 - A_3)(A_3 - A_1 + A_2) + \\ + p^2 A_1 (A_1 - A_2)(A_3 - A_1 - A_2) \end{bmatrix} \\ r\begin{bmatrix} p^2 A_1 (A_3 - A_1)(A_1 - A_2 + A_3) + \\ + q^2 A_2 (A_2 - A_3)(A_1 - A_2 - A_3) \end{bmatrix} \end{pmatrix}. \quad (8)$$

accurate to a first-order infinitesimal ε.

We consider this expression only in the first approximation. The equations of controlled motion (1) simplified on the basis of expression (8) in projections on the major central axes of inertia have the form:

$$A_1\dot{p} + \left(A_3 - A_2\right)qr = -b\frac{A_1 p}{G} - \lambda A_1 p +$$
$$+\frac{P}{A_1 A_2 A_3}p\left[q^2 A_2 (A_1 - A_2)(A_2 - A_3 + A_1) + \right.$$
$$\left. + r^2 A_3 (A_1 - A_3)(A_3 - A_2 + A_1)\right],$$
$$A_2\dot{q} + \left(A_1 - A_3\right)pr = -b\frac{A_2 q}{G} - \lambda A_2 q +$$
$$+\frac{P}{A_1 A_2 A_3}q\left[r^2 A_3 (A_2 - A_3)(A_3 - A_1 + A_2) + \right.$$
$$\left. + p^2 A_1 (A_2 - A_1)(A_1 - A_3 + A_2)\right],$$
$$A_3\dot{r} + \left(A_2 - A_1\right)pq = -b\frac{A_3 r}{G} - \lambda A_3 r +$$
$$+\frac{P}{A_1 A_2 A_3}r\left[p^2 A_1 (A_3 - A_1)(A_1 - A_2 + A_3) + \right.$$
$$\left. + q^2 A_2 (A_3 - A_2)(A_2 - A_1 + A_3)\right]. \quad (9)$$

The kinematic relations are omitted because Eqs. (9) form a closed system. These equations are further analyzed.

3. SOLUTION OF THE OPTIMAL DECELERATION PROBLEM

Let us note that the torque exerted by viscous fluid in the cavity is internal, while the torque of the linear drag of the medium is external. Multiplying the first equation in (8) by G_1, the second equation by G_2, and the third equation by G_3, and summing them, we obtain a scalar equation to be integrated

$$\dot{G} = -b\left(t, G\right) - \lambda G, \; G(t_0) = G^0. \quad (10)$$

Upon solving Cauchy problem (10), we obtain from the condition of stopping the rotation (5) the required expression for the time $T = T(t_0, \boldsymbol{G}_0)$ and the Bellman function $W(t, \boldsymbol{G}) = T(t, \boldsymbol{G})$.

Recall that $\boldsymbol{G} = J\boldsymbol{\omega}$.

In the general case, for an arbitrary function $b = b(t, \boldsymbol{G})$ in (10), the analytical integration of the Cauchy problem is complicated; however, it can be solved numerically. Equations (10) imply that the evolution of the magnitude of the kinetic moment $\boldsymbol{G}$ is affected by the control moment and the medium drag. The internal torque of the viscous fluid in the cavity has no effect. If $b = b(t)$ (i.e., the function $b(t)$ is independent of $\boldsymbol{G}$), we obtain the solution of boundary problem (10)

$$G(t) = G^0 \exp\left(-\lambda\left(t - t_0\right)\right) - \int_{t_0}^{t} b(\tau)\exp\left(-\lambda(t - \tau)\right)d\tau, \quad (11)$$

where $G^0 = \exp\left(-\lambda t_0\right)\int_{t_0}^{T} b(\tau)\exp\left(\lambda\tau\right)d\tau$

According to (4), Eq. (11) is solvable with respect to the unknown T, which leads to the construction of the time-optimal solution. Here, t is the current time of deceleration and T is the optimal time. For $b = \text{const}$ and $t_0 = 0$, the solutions of equation (2.1) and boundary problem (11) are written as

$$G(t) = \frac{1}{\lambda}\left[\left(G^0\lambda + b\right)\exp(-\lambda t) - b\right], \quad (12)$$

$$T = \frac{1}{\lambda}\ln(G^0\frac{\lambda}{b}+1).$$

Next, we consider in detail case (12). Let us multiply the first equation in (8) by p, the second equation by q, the third equation by r, and sum the results. The resulting expression for the derivative of the kinetic energy H is

$$\dot{H} = -\frac{2bH}{G^2} - 2\lambda H + \frac{P}{A_1A_2A_3}\Big[p^2q^2(A_1-A_2)^2 \times$$
$$\times(A_3 - A_1 - A_2) + p^2r^2(A_1-A_3)^2(A_2-A_1-A_3)+$$
$$+q^2r^2(A_2-A_3)^2(A_1-A_2-A_3)\Big].(13)$$

Consider an undisturbed motion ($b = \lambda = \varepsilon = 0$). In the absence of perturbations, the rotation of the rigid body is a Euler–Poinsot motion. The variables G and H become constant and φ, ψ, and θ are functions of time t. The slow variables in the perturbed motion are G and H, and the fast variables are the Euler angles φ, ψ, and θ.

Consider a motion under the condition $2HA_1 \geq G^2 \geq 2HA_2$ corresponding to the trajectories of the kinetic moment vector, which envelope the major torque axis Oz_1. Define

$$k^2 = \frac{(A_2-A_3)(2HA_1-G^2)}{(A_1-A_2)(G^2-2HA_3)} \quad (0 \leq k^2 \leq 1), \ (14)$$

which is the module of elliptic functions describing this motion and is a function of the kinetic moment G and the kinetic energy H (in the case of unperturbed motion, it is a constant).

To construct the averaged first-approximation system of equations, we substitute the solution of the unperturbed Euler–Poinsot motion into the right-hand side of Eq. (13) and average over the variable ψ and then over time t taking into account the dependences of φ and θ on t. Here, we retain the notation for the slow variables G and H. As a result, we obtain

$$\frac{dH}{dt} = -\frac{2bH}{G} - 2\lambda H -$$
$$-\frac{4PH^2(A_1-A_3)(A_1-A_2)(A_2-A_3)}{3A_1^2A_2^2A_3^2S^2(k)}\times$$
$$\times\Big\{A_2(A_1-A_3)(A_1+A_3-A_2)\Big[k^2V(k)-U(k)\Big]+$$
$$+A_1(A_2-A_3)(A_3+A_2-A_1)\Big[(k^2-2)U(k)+k^2\Big]+$$
$$+A_3(A_1-A_2)(A_1+A_2-A_3)\Big[(1-2k^2)U(k)+k^2\Big]\Big\},(15)$$

where $U(k) = 1 - E(k)/K(k)$, $V(k) = 1 + E(k)/K(k)$,

$$S^2(k) = \Big[A_2 - A_3 + (A_1 - A_2)k^2\Big]^2.$$

Here, $K(k)$ and $E(k)$ are the complete elliptic integrals of the first and second kind, respectively [13]. Equation (15) implies that the resistance of the medium and the torque of the viscous fluid in the body cavity as well the control moment cause the evolution of the kinetic energy H of the body. The expression in the braces on the right-hand side of equation (15) is positive (for $A_1 > A_2 > A_3$) because of the inequalities $(1 - k_2)K \leq E \leq K$ (see [13]). Consequently, $dH/dt < 0$ because $H > 0$; i.e., H is a strictly decreasing variable for any $k^2 \in [0, 1]$. Note that Eq.(15) has an essential singularity as $G \to 0$.

Differentiating expression (14) for k^2 with regard to (15), we obtain a the differential equation

$$\frac{dk^2}{dt} = \frac{PG^2(A_1-A_3)\big[A_2(A_1+A_3-A_2)+2A_1A_3\big]}{3A_1^2A_2^2A_3^2}\times$$
$$\times\left\{(1-\chi)(1-k^2) - [(1-\chi)+(1+\chi)k^2]\frac{E(k)}{K(k)}\right\},(16)$$

where $\chi = \dfrac{3A_2[(A_1^2+A_3^2)-A_2(A_1+A_3)]}{(A_1-A_3)[A_2(A_1+A_3-A_2)+2A_1A_3]}$.

Equations (15) and (16) were obtained by the method of averaging [1, 2, 7]. This corresponds to the fact that the kinetic energy of the body rotation is much greater than the control vector magnitude, the resistance of the medium is assumed to be weak of the infinitesimal order ε, and the cavity is filled with highly viscous fluid.

The value $k^2 = 1$ is associated with the equality $2HA_2 = G^2$, which corresponds to the separatrix of the Euler–Poinsot motion. Equation (16) describes the averaged motion of the endpoint of the kinetic moment vector $\boldsymbol{G}$ on a sphere of radius G. Notice that the evolution of k^2 is affected only by the torque of the viscous fluid in the cavity, and, because this equation is integrated independently, the influence of the torque of the viscous fluid in the cavity, the control moment, and the resistance moment is partially separated. An analysis of Eq. (16) shows that there are no stationary values of k except for $k = 0$ and $k = 1$.

4. NUMERICAL CALCULATION

4.1 The dimensionless differential equation of the motion

We reduce Eq.(15) and (16) and the differential equation for $b = \text{const}$ the kinetic moment for to a dimensionless form. As the characteristic parameters of the problem, we use the value of the kinetic moment at the initial time $G_0 = G(t_0)$ and the time T (12)

$$\tilde{G} = GG_0^{-1}, \ \tilde{t} = tT^{-1}.$$

The value of the dimensionless kinetic energy is defined (see [1]) as

$$\tilde{H} = 2HA_1G_0^{-2}.$$

We obtain a dimensionless system in the form

$$\frac{d\tilde{G}}{d\tilde{t}} = -\left(\frac{b}{G_0} + \lambda\tilde{G}\right)T, \tag{17}$$

$$\frac{dk^2}{d\tilde{t}} = \frac{PT\tilde{G}^2G_0^2\left(A_1 - A_3\right)}{3A_1^3A_2^2A_3^2} \times$$

$$\times\left\{(1-\chi)(1-k^2) - [(1-\chi) + (1+\chi)k^2]\frac{E(k)}{K(k)}\right\},$$

$$\frac{d\tilde{H}}{d\tilde{t}} = -T\left(\frac{2b\tilde{H}}{\tilde{G}G_0} + 2\lambda\tilde{H} + \right.$$

$$+\frac{4PG_0^2\tilde{H}^2(A_1-A_3)(A_1-A_2)(A_2-A_3)}{3A_1^3A_2^2A_3^2S^2\left(k\right)} \times$$

$$\times\left\{A_2(A_1-A_3)(A_1+A_3-A_2)\left[k^2V(k) - U(k)\right] + \right.$$

$$+A_1(A_2-A_3)(A_3+A_2-A_1)\left[(k^2-2)U(k) + k^2\right] +$$

$$\left.\left. +A_3(A_1-A_2)(A_1+A_2-A_3)\left[(1-2k^2)U(k) + k^2\right]\right\}\right).$$

Here, we performed averaging because expressions (13) and (14) imply that H and k^2 are slow variables.

4.2 A numerical integration of the system

We make a numerical integration of system (17) over the interval [0, 1], which corresponds to complete body deceleration. The initial function values for this calculation were $\hat{G}(0) = G_0 = 1$, $\hat{H}(0) = 1$, and $k^2(0) \approx 1$. The moments of inertia have the values (see [1]): $A_1 = 8$, $A_2 = 6$, and $A_3 = 4$. The calculations were performed for various values of λ, b, and P, which makes it possible to study the influence of different force factors on the character of the rigid body deceleration. For each case, we first calculated the deceleration time and then the characteristics of the body motion in the corresponding time interval.

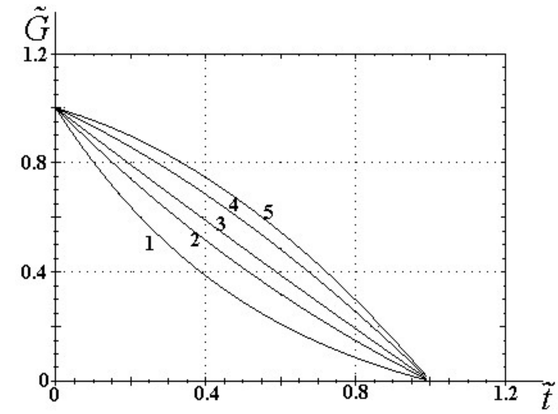

Fig. 1 – Changing magnitude of the kinetic moment for different λ

Figures 1 and 2 illustrate the numerical analysis for $P = 10^{-1}$, $b = 10^{-1}$, and $\lambda = 0.5$, 10^{-1}, 10^{-2} (curves 1, 2, and 3, respectively). It is seen that the decrease in the moment of medium resistance forces leads to a decreased gradient of the body deceleration and an almost linearly dependence of the kinetic moment (curve 3 in Fig. 1) Figures 3 and 4 show the results of calculations for $P = 10^{-1}$, $\lambda = 10^{-1}$, and $b = 10^{-2}$, $5{\bullet}10^{-2}$, $5{\bullet}10^{-1}$ (curves 1, 2, and 3, respectively).

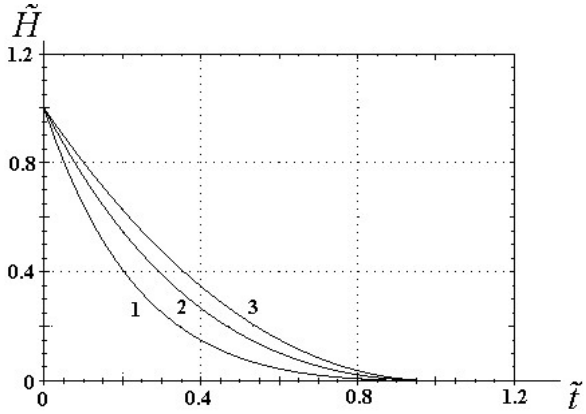

Fig. 2 – Changing magnitude of the kinetic energy for different λ

It is seen that the increase in the moment of control forces (curve 3 in Fig. 4) leads to a speedup of the body deceleration and an almost linearly changing magnitude of the kinetic moment at large values of b (curve 3 in Fig. 3).

The change of P from 1 to 10^{-2} has no effect on the general behavior of the functions $\hat{G} = \hat{G}(t)$ and $\hat{H} = \hat{H}(t)$ because the torque of the viscous fluid in the cavity does not appear in the first equation of system (17) and its effect on the change of the kinetic energy is smaller than the influence of the moment of resistance forces and the control moment.

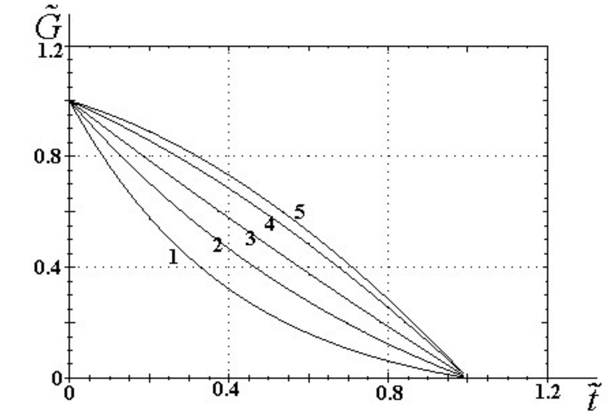

Fig. 3 – Changing magnitude of the kinetic moment for different *b*

The numerical results show that, for the values of λ, *b*, and *P* indicated above, the module of elliptic functions k^2 insignificantly decreases from around 1 to 0.9996.

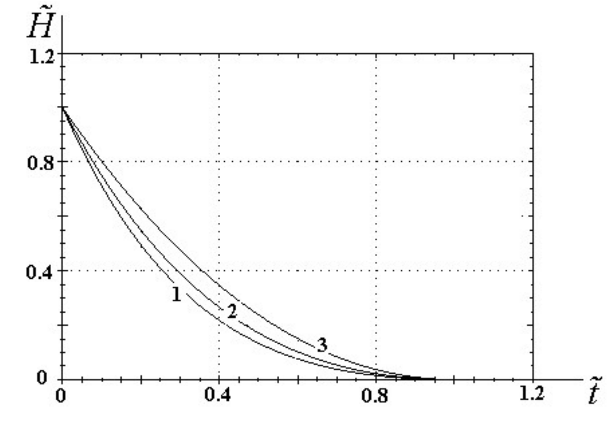

Fig. 4 – Changing magnitude of the kinetic energy for different *b*

5. CONCLUSION

The problem of time-optimal deceleration of rotations of a dynamically nonsymmetric quasirigid body in a resistive medium was studied analytically and numerically. The asymptotic approach made is possible to determine the control, evolutions of the square of the magnitude of the elliptic functions modulus k^2, and dimensionless kinetic energy and kinetic moment. The qualitative properties of the optimal motion were found.

According to numerical computation for asymmetric body under the action of the torques of forces of a viscous fluid in a cavity showed that the behavior of the kinetic energy function depends on the ratio of dimensionless coefficients characterizing these disturbing torques, and also depends on the characteristics of controlled motion. The deceleration of the rigid body occurs, when essential influence of the torque of a viscous liquid is.

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АКТИВНЕ ГАЛЬМУВАННЯ ОБЕРТАЛЬНИХ РУХІВ ДИНАМІЧНО НЕСИМЕТРИЧНОГО КВАЗИТВЕРДОГО ТІЛА

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Досліджено задачу оптимального за швидкодією гальмування обертань динамічно несиметричного твердого тіла. На тверде тіло діє гальмуючий момент сил лінійного опору середовища. Керування обертаннями проводиться за допомогою моменту сил, обмеженого за модулем. Визначено оптимальний закон керування для гальмування обертань твердого тіла у формі синтезу і фазові траєкторії. Керований рух являє собою рух типу Ейлера-Пуансо із змінною за часом величиною кінетичного моменту тіла.

Ключові слова: оптимальне гальмування, середовище з опором, асиметричне тіло, порожнина.

АКТИВНОЕ ТОРМОЖЕНИЕ ВРАЩАТЕЛЬНЫХ ДВИЖЕНИЙ ДИНАМИЧЕСКИ НЕСИММЕТРИЧНОГО КВАЗИТВЕРДОГО ТЕЛА

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Исследована задача оптимального по быстродействию торможения вращений динамически несимметричного твердого тела. На твердое тело действует тормозящий момент сил линейного сопротивления среды. Управление вращениями производится с помощью момента сил, ограниченного по модулю. Определены оптимальный закон управления для торможения вращений твердого тела в форме синтеза и фазовые траектории. Управляемое движение представляет собой движение типа Эйлера-Пуансо с изменяющейся по времени величиной кинетического момента тела.

Ключевые слова: оптимальное торможение, сопротивляющаяся среда, асимметричное тело, полость.

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