

**O.Yu. Khetselius, d.ph.-m.n., A.A. Svinarenko, c.ph.-m.n.**

*Одесский государственный экологический университет*

## **ANALYSIS OF THE FRACTAL STRUCTURES IN WAVE PROCESSES**

*On the basis of wavelet analysis and multifractal formalism it has been carried out an analysis of fractal structures in the wave processes (capillary-gravitational ripple).*

**Key words:** *fractals structures, wave processes*

**Introduction.** In last years it is of a great importance the experimental and theoretical studying of the non-linear dynamical systems with aim to discover the fractal features and elements of dynamical chaos (e.g. [1-18]). One of the effective approaches to solving such a problem is the multifractal and wavelet analyses. The foundations and application information on the continuous wavelet transform-based method of multifractal analysis are presented in Ref. [12]. An extension of the concept of multifractals to irregular functions through the use of wavelet transform modulus maxima and potential and limitations of the multifractal formalism in the study of non-stationary processes and short signals are in details considered in these references. Especial attention is turned to the multifractality loss effects in the dynamics of different types of systems. A review of fundamental results on the manifestation of fractal structure in wave processes is presented in [1]. Some patterns of the wave scattering and radiation by fractal structures are examined. Principal methods of random signal analysis are described to reveal different fractal structures associated with these signals and data on the wave field fractal properties are widely discussed in literature. One of the attractive examples of the fractal structures in the wave fields is the fractal ones in the wind waves on the sea surface. As it is indicated in many references (e.g. [1]) here it is possible an effect of the rays chaos. It has been defined that the fractal features are available in the shape of the disturbed sea surface as for determinative as random waves. In many papers (e.g. [1-4,13,14]) the flow exponential spectra of the wind waves with exponent  $11/3$  and  $4$  for gravitate waves are used (for capillary waves it is equal to  $17/6$ ). In fact, here the rang of scales of the fractal behavior is limited by the distortion correlation radius. It is important to note that a scattering of the waves on the sea surface can be represented as a scattering on the fractals. In paper by Elgar and Mayer-Kress (see Refs. in [1]) another approach is used in studying the fractal properties of the distortion. In fact a dimension of the attractor in the phase space is defined by the Tackens algorithm. It is shown that the surface distortion is not governed by finite dimension dynamical system with a strange attractor. In papers of Zaslavsky et al (e.g. [5]) the fractal properties of the sea surface have been considered on the scales which are more than the distortion correlation radius. In particular, on the basis of analyzing the aero-photo images it has been found the fractality in distribution of the zones for waves falling ( $d=0,5$ ). In paper by Naugolnyh-Zosimov (see Refs. in [1]) the fractal properties of the sea surface have been considered too and the laser scanning locator measurements of distribution of the mirror dots along space-temporal line, defined by the vessel running. The cited measurements were carried out in the tropical Atlantic in the opened ocean, where the tropical passates provided the stationary developed distortion during several days. In fact, multi-hours data for intensity of the capillary ripple are received. In our paper we have carried out multifractal analysis of some patterns of the wave scattering, in particular, non-linear dynamical effects in the indicated wave processes. The spatial spectrum of the large scaled changing intensity of the capillary-gravitational ripple according to the laser scanning data has been studied.

**Method.** Let us further consider wavelet analysis and multi-fractal formalism, following Refs. [11,15-18]. Since last decades, many scientists use the new powerful tool based on the wavelet decomposition for analyzing various signals. At present, the family of analyzing function dubbed wavelets is being increasingly used in problems of pattern recognition; in processing and synthesizing various signals; in analysis of images of any kind (X-ray picture of a kidney, an image of mineral, etc.); for study of turbulent fields, for contraction (compression) of large volumes of information, and in many other cases. Wavelets are fundamental building block functions, analogous to the sine and cosine functions. Fourier transform extracts details from the signal frequency, but all information about the location of a particular frequency within the signal is lost. At the expense of their locality the wavelets have advantages over Fourier transform when non-stationary signals are analyzed. Here, we use non-decimated wavelet transform that has temporal resolution at coarser scales.

The dilation and translation of the mother wavelet  $\psi(t)$  generates the wavelet as follows:  $\psi_{j,k}(t) = 2^{j/2}\psi(2^j t - k)$ . The dilation parameter  $j$  controls how large the wavelet is, and the translation parameter  $k$  controls how the wavelet is shifted along the  $t$ -axis. For a suitably chosen mother wavelet  $\psi(t)$ , the set  $\{\psi_{j,k}\}_{j,k}$  provides an orthogonal basis, and the function  $f$  which is defined on the whole real line can be expanded as

$$f(t) = \sum_{k=-\infty}^{\infty} c_{0k} \varphi_{0,k}(t) + \sum_{j=1}^J \sum_{k=-\infty}^{\infty} d_{jk} \psi_{j,k}(t), \quad (1)$$

where the maximum scale  $J$  is determined by the number of data, the coefficients  $c_{0k}$  represent the lowest frequency smooth components, and the coefficients  $d_{jk}$  deliver information about the behavior of the function  $f$  concentrating on effects of scale around  $2^{-j}$  near time  $k \times 2^{-j}$ . This wavelet expansion of a function is closely related to the discrete wavelet transform (DWT) of a signal observed at discrete points in time. In practice, the length of the signal, say  $n$ , is finite and, for our study, the data are available monthly, i.e. the function  $f(t)$  in Eq. (1) is now a vector  $f = (f(t_1), \dots, f(t_n))$  with  $t_i = i/n$  and  $i = 1, \dots, n$ . With these notations, the DWT of a vector  $f$  is simply a matrix product  $d = Wf$ , where  $d$  is an  $n \times 1$  vector of discrete wavelet coefficients indexed by 2 integers,  $d_{jk}$ , and  $W$  is an orthogonal  $n \times n$  matrix associated with the wavelet basis. For computational reasons, it is simpler to perform the wavelet transform on time series of dyadic (power of 2) length. One particular problem with DWT is that, unlike the discrete Fourier transform, it is not translation invariant. This can lead to Gibbs-type phenomena and other artefacts in the reconstruction of a function. The non-decimated wavelet transform (NWT) of the data  $(f(t_1), \dots, f(t_n))$  at equally spaced points  $t_i = i/n$  is defined as the set of all DWT's formed from the  $n$  possible shifts of the data by amounts  $i/n$ ;  $i = 1, \dots, n$ . Thus, unlike the DWT, there are  $2^j$  coefficients on the  $j$ th resolution level, there are  $n$  equally spaced wavelet coefficients in the NWT:  $d_{jk} = n^{-1} \sum_{i=1}^n 2^{j/2} \psi[2^j(i/n - k/n)] y_i$ ,  $k = 0, \dots, n-1$ , on each resolution level  $j$ . This results in  $\log_2(n)$  coefficients at each location. As an immediate consequence, the NWT becomes translation invariant. Due to its structure, the NWT implies a finer sampling rate at all levels and thus provides a better exploratory tool for analyzing changes in the scale (frequency) behavior of the underlying signal in time. These advantages of the NWT over the DWT in time series analysis are demonstrated in Nason et al (e.g.[12]). As in the Fourier domain, it is important to assess the power of a signal at a given resolution. An evolutionary wavelet spectrum (EWS) quantifies the contribution to process variance at the scale  $j$  and time  $k$ . From the above paragraphs, it is easy to plot any time series into the wavelet domain. Another way of viewing the result of a NWT is to represent the temporal evolution of the data at a given scale. This type of representation is very useful to com-

pare the temporal variation between different time series at given scale. To obtain the results, smooth signal  $S_0$  and the detail signals  $D_j$  ( $j=1, \dots, J$ ) are

$$S_0(t) = \sum_{k=-\infty}^{\infty} c_{0k} \Phi_{0,k}(t) \quad \text{and} \quad D_j(t) = \sum_{k=-\infty}^{\infty} d_{jk} \Psi_{j,k}(t). \quad (2)$$

The fine scale features (high frequency oscillations) are captured mainly by the fine scale detail components  $D_J$  and  $D_{J-1}$ . The coarse scale components  $S_0$ ,  $D_1$ , and  $D_2$  correspond to lower frequency oscillations of the signal. Note that each band is equivalent to a band-pass filter. Further we use the Daubechies wavelet as mother wavelet [12]. This wavelet is bi-orthogonal and supports discrete wavelet transform. Using a link between wavelets and fractals, one could make calculating the multi-fractal spectrum. As usually, the homogeneous fractals are described by single fractal dimension  $D(0)$ . Non-homogeneous or multifractal objects are described by spectrum  $D(q)$  of fractal dimensions or multifractal spectrum. A problem of its calculation reduces to definition of singular spectrum  $f(\alpha)$  of measure  $\mu$ . It associates Hausdorff dimension and singular indicator  $\alpha$ , that allows calculating a degree of singularity:  $N_\alpha(\varepsilon) \sim \varepsilon^{-f(\alpha)}$ . Below we use a formalism, which allows defining spectra of singularity and fractal dimension without using standard Legendre transformations. This idea at first used in ref.[8]. Wavelet transformation of some real function  $F$  can be also defined as

$$W_\Psi[F](b, a) = (1/a) \int_{-\infty}^{+\infty} F(x) \Psi\left(\frac{x-b}{a}\right) dx, \quad (3)$$

where parameter  $b$  denotes a shift in space (a space scale). The analyzing splash  $\Psi$  has to be localized as in space as on frequency characteristics. The most correct way of estimate of the function  $D(h), f(\alpha)$  is in analysis of changing a dependence of the distribution function  $Z(q, a)$  on modules of maximums of the splash-transfers under scale changes

$$Z = \sum_{i=1}^{N(a)} (\omega_i(a))^q, \quad (4)$$

where  $I=1, \dots, N(a)$ ;  $N(a)$  is a number of localized maximums of transformation  $W_\Psi[F](b, a)$  for each scale  $a$ ; function  $\omega(a)$  can be defined in terms of coefficients of the splash-transformations as

$$\omega_i(a) = \max_{\substack{(x, a') \in L \\ a' < a}} |W_\Psi[F](x, a')|, \quad (5)$$

where  $l_i \in L(a)$ ;  $L(a)$  is a set of such lines, which make coupling the splash-transformation coefficient maximums (they reach or make cross-section of a level, which is corresponding to scale  $a$ ). In the limit  $a \rightarrow 0^+$  the distribution function  $Z(q, a)$  manifests the behaviour, which is corresponding to a degree law:  $Z(q, a) \sim a^{-\tau(q)}$ . To calculate a singularity spectrum, the standard canonical approach can be used. It is based on using such functions:

$$h(a, q) = \frac{1}{Z(a, q)} \frac{\partial Z(a, q)}{\partial q}, \quad (6a)$$

$$\frac{\partial Z}{\partial q} = \sum_{i=1}^{N(a)} \omega_i(a)^q \ln \omega_i(a), \quad (6b)$$

$$D(a, q) = qh(a, q) - \ln Z(a, q). \quad (6c)$$

The spectra  $D(q)$  and  $h(q)$  are defined by standard way as follows:

$$D(q) = \lim_{a \rightarrow 0} \frac{D(a, q)}{\ln a}, \quad (7a)$$

$$h(q) = \lim_{a \rightarrow 0} \frac{h(a, q)}{\ln a}. \quad (7b)$$

Other details can be found in Refs. [11,15-18].

**Results and conclusions.** Using the above described formalism, we have carried out a multifractal analysis of spatial spectrum of the large scaled changing intensity of the capillary-gravitational ripple. In fig.1 it is presented the initial spatial spectrum the large scaled changing intensity of the capillary-gravitational ripple according to the laser scanning data [19].

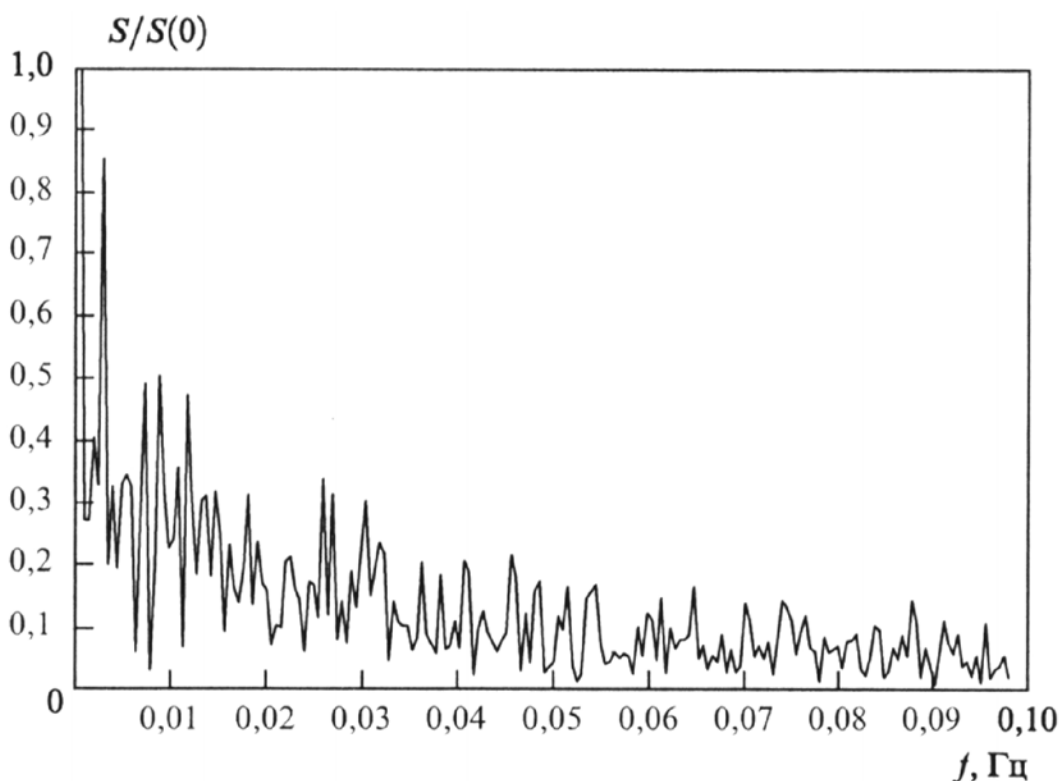


Fig. 1 - The initial spectrum of the large scaled changing intensity of the capillary-gravitational ripple according to the laser scanning data [19]. In fact a number of reflections under scanning a surface by the thin laser flux (the vessel velocity is 8m/s; frequency 1 Gz is corresponding to spatial scale 8 m).

Using the PC complex “Geomath” (c.f.[15]) we have performed the numerical calculations of the fractal spectrum for the capillary-gravitational ripple. The numerical estimates have shown that the fractals dimensions are lying in the interval [0,65-0,88]. These data are satisfactory agreed with the preliminary estimates within the correlation integral formalism [11]. Our analysis confirms the universal conclusion regarding availability of the fractal features for distortion in large scales to a weak wave turbulence for waves with non-decay spectrum. In fact speech is about a model, which describes a growth and stationary spectra of the wave distortion.

### List of References

1. Zosimov V.V., Lyamshev L.M. Fractals in wave processes//Phys. Uspekhi.-1995.-Vol.38.-P.347-380.
2. Mandelbrot B. Fractal geometry of nature.- M.: Mir, 2002.-656p.
3. Schertzer D., Lovejoy S. Fractals: Physical Origin and Properties, Ed. Peitronero L.-N.-Y.: Plenum Press.-1990.-P.71-92.
4. Sprott J.C., Vano J.A., Wildenberg J.C., Anderson M.B., Noel J.K. Coexistence and chaos in complex ecologies // Phys. Lett. A.-2005.-V.335, № 2-3.-P.207-212.
5. Zaslavsky G.M. Stochasticity of dynamical systems.- Moscow: Nauka.-1998.
6. Grassberger P., Procaccia I. Measuring the strangeness of strange attractors// Physica D.-1983.-Vol.9.-P.189-208.
7. Sivakumar B. Chaos theory in geophysics: past, present and future // Chaos, Solitons & Fractals.-2004.-Vol.19.-P.441-462.
8. Kaplan J.L., Yorke J.A. Chaotic behavior of multidimensional difference equations // Functional differential equations and approximations of fixed points. Springer Lecture Notes in Mathematics No.730 / H.-O. Peitgen, H.-O. Walter (Eds.).- Berlin: Springer, 1979.-P.204-227.
9. Packard N.H., Crutchfield J.P., Farmer J.D., Shaw R.S. Geometry from a time series// Phys. Rev. Lett.-1980.-Vol.45.-P.712-716.
10. Schreiber T. Interdisciplinary application of nonlinear time series methods // Phys. Rep.-1999.-Vol.308.-P. 1-64.
11. Svinarenko A.A. Regular and chaotic dynamics of multi-oscillator dynamical systems// Photoelectronics.-2002.-Vol.11.-P.81-84.
12. Daubechies I. Ten Lectures on Wavelets.- Philadelphia: SIAM, 1992.- 380p.
13. Morlet J., Arens G., Fourgeau E. and Giard D. Wave propagation and sampling theory// Geophysics.-1982.-Vol.47.-p.203-236.
14. Nason G., von Sachs R., Kroisand G. Wavelet processes and adaptive estimation of the evolutionary wavelet spectrum // J.Royal Stat.Soc. -2000.-Vol.B62.-P.271-292.
15. Glushkov A.V., Khokhlov V.N., Svinarenko A.A., Bunyakova Yu.Ya., Prepelitsa G.P. Wavelet analysis and sensing the total ozone content in the earth atmosphere: Micros technology "Geomath"//Sensor Electr. and Microsys.Techn.-2005.-Vol.2, N3.-P.51-60.
16. Glushkov A.V., Khokhlov V.N., Tsenenko I.A. Atmospheric teleconnection patterns: wavelet analysis// Nonlin. Proc.in Geophys.-2004.-Vol.11,N3.-P.285-293.
17. Khokhlov V.N., Glushkov A.V., Loboda N.S., Bunyakova Yu.Ya. Short-range forecast of atmospheric pollutants using non-linear prediction method// Atmospheric Environment (Elsevier).-2008.-Vol.42.-P. 7284-7292.
18. Glushkov A.V., Loboda N.S., Khokhlov V.N., Lovett L. Using non-decimated wavelet decomposition to analyse time variations of North Atlantic Oscillation, eddy kinetic energy, and Ukrainian precipitation // Journal of Hydrology (Elsevier).-2006.-Vol. 322,N1-4.-P.14-24.

#### **Аналіз фрактальних структур у хвильових процесах. Хецеліус О.Ю., Свиначенко А.А.**

Виконано аналіз фрактальних структур у хвильових процесах (капілярно - гравітаційні хвилі) на підставі вейвлет - аналізу та мультифрактального формалізму.

**Ключові слова:** фрактальні структури, хвильові процеси

#### **Анализ фрактальных структур в волновых процессах. Хецеліус О.Ю., Свиначенко А.А.**

Выполнен анализ фрактальных структур в волновых процессах (капиллярно-гравитационные волны) на основе вейвлет - анализа и мультифрактального формализма.

**Ключевые слова:** фрактальные структуры, волновые процессы